

**DEPARTMENT OF ELECTRICAL ENGINEERING**

**BASIC ELECTRICAL ENGINEERING (4 credit)**

**Course Code: 23UEEL1101**

**Unit-I D.C. Circuits (08 HOURS)**

**1.1 Kirchoff's Laws:-**

**1.1.1. Kirchoff's current law or point law (KCL)**

Statement:- In any electrical network, the algebraic sum of the currents meeting at a point is zero.

$\Sigma I = 0$ .....at a junction or node

Assumption:- Incoming current = positive

Outgoing current = negative

**1.1.2. Kirchoff's voltage law or mesh law (KVL)**

Statement:- The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emfs in that path is zero.

$\Sigma IR + \Sigma emf = 0$ ..... round the mesh

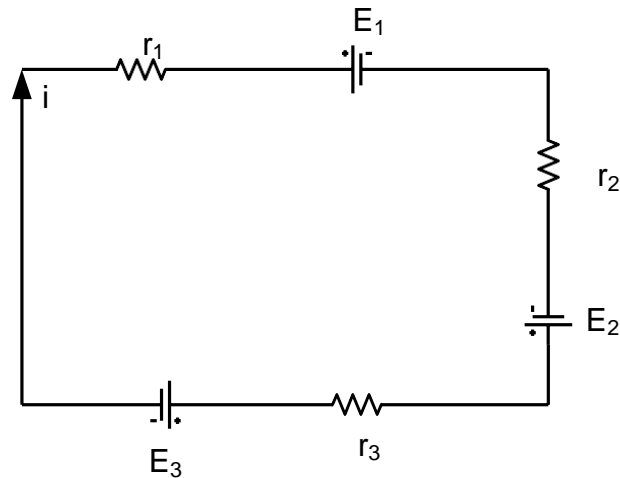
Assumption:- i) Rise in voltage (If we go from negative terminal of the battery to positive terminal) = positive

ii) Fall in voltage (If we go from positive terminal of the battery to negative terminal) = negative

iii) If we go through the resistor in the same direction as current then there is a fall in potential. Hence this voltage is taken as negative.

iv) If we go through the resistor against the direction of current then there is a rise in potential. Hence this voltage drop is taken as positive.

**Example:-** Write the loop equation for the given circuit below  
(Supplementary exam 2004)



Solution: Apply KVL to the loop,

$$-ir_1 - E_1 - ir_2 + E_2 - ir_3 - E_3 = 0$$

$$\Rightarrow E_1 - E_2 + E_3 = -ir_1 - ir_2 - ir_3$$

$$\Rightarrow E_1 - E_2 + E_3 = -i(r_1 + r_2 + r_3)$$

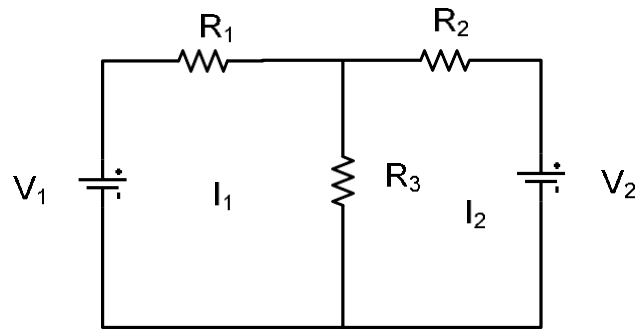
## 1.2. MAXWELL'S LOOP CURRENT METHOD (MESH ANALYSIS)

Statement:- This method determines branch currents and voltages across the elements of a network. The following process is followed in this method:-

- Here, instead of taking branch currents (as in Kirchoff's law) loop currents are taken which are assumed to flow in the clockwise direction.
- Branch currents can be found in terms of loop currents
- Sign conventions for the IR drops and battery emfs are the same as for Kirchoff's law.
- This method is easier if all the sources are given as voltage sources. If there is a current source present in a network then convert it into equivalent voltage source.

**Explanation:-**

Consider a network as shown in Fig. below. It contains two meshes. Let  $I_1$  and  $I_2$  are the mesh currents of two meshes directed in clockwise.



Apply KVL to mesh-1,

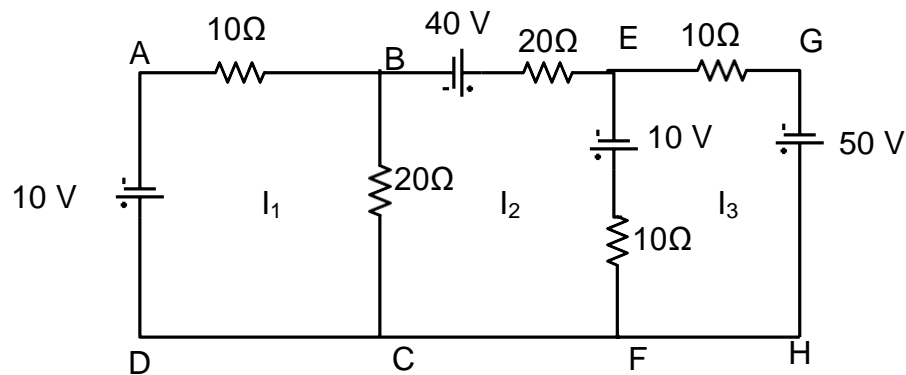
$$V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

Apply KVL to mesh-2,

$$-I_2 R_2 - V_2 - (I_2 - I_1) R_3 = 0$$

When we consider mesh-1, the current  $I_1$  is greater than  $I_2$ . So, current through  $R_3$  is  $I_1 - I_2$ . Similarly, when we consider mesh-2, the current  $I_2$  is greater than  $I_1$ . So, current through  $R_3$  is  $I_2 - I_1$ .

**Example:** Find  $I_1$ ,  $I_2$  and  $I_3$  in the network shown in Fig below using loop current method



Solution:- For mesh ABCDA,

$$\begin{aligned}
 & -I_1 \times 10 - (I_1 - I_2) \times 20 - 10 = 0 \\
 \Rightarrow & 3I_1 - 2I_2 = -1 \qquad (1)
 \end{aligned}$$

For mesh BEFCB,

$$\begin{aligned}
 & 40 - I_2 \times 20 + 10 - (I_2 - I_3) \times 10 - (I_2 - I_1) \times 20 = 0 \\
 \Rightarrow & 2I_1 - 5I_2 + I_3 = -5 \qquad (2)
 \end{aligned}$$

For mesh EGHFE,

$$\begin{aligned}
 & -10I_3 + 50 - (I_3 - I_2) \times 10 - 10 = 0 \\
 \Rightarrow & I_2 - 2I_3 = -4 \qquad (3)
 \end{aligned}$$

Equation (2) x 2 + Equation (3)

$$4I_1 - 9I_2 = -14 \qquad (4)$$

Solving eq<sup>n</sup> (1) & eq<sup>n</sup> (4)

$$I_1 = 1 \text{ A}, I_2 = 2 \text{ A}, I_3 = 3 \text{ A}$$

### 1.3. NODAL ANALYSIS

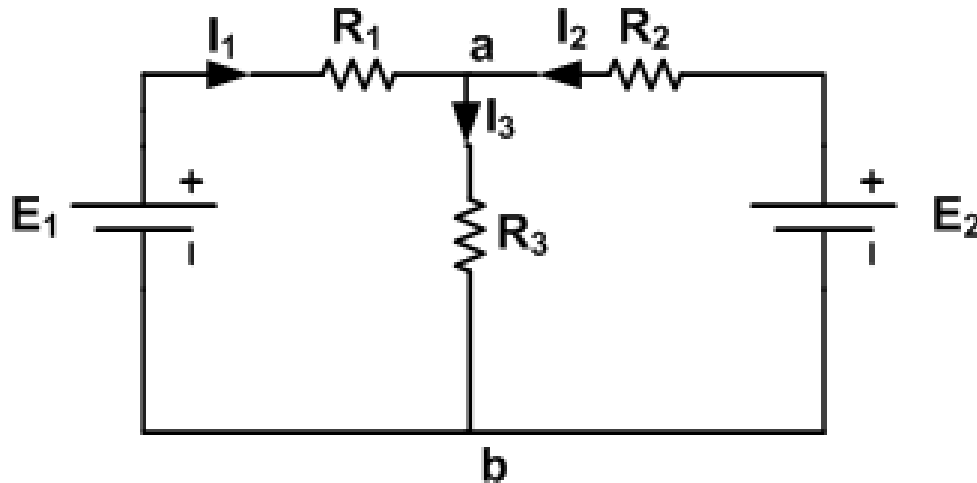
**Statement:-** This method determines branch currents in the circuit and also voltages at individual nodes.

The following steps are adopted in this method:-

- Identify all the nodes in the network.
- One of these nodes is taken as reference node in at zero potential
- The node voltages are measured w.r.t the reference node
- KCL to find current expression for each node
- This method is easier if all the current sources are present. If any voltage source is present, convert it to current source

- The number of simultaneous equations to be solved becomes (n-1) where 'n' is the number of independent nodes.

**Explanation:-**



At node 'a'  $I_1 + I_2 = I_3$

By ohms law,  $I_1 = \frac{E_1 - V_a}{R_1}, I_2 = \frac{E_2 - V_a}{R_2}, I_3 = \frac{V_a}{R_3}$

Therefore,  $V_a \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$

or,  $V_a \begin{bmatrix} \frac{1}{R_1} & \frac{1}{R_2} & \frac{1}{R_3} \\ 1 & 2 & 3 \end{bmatrix} - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$

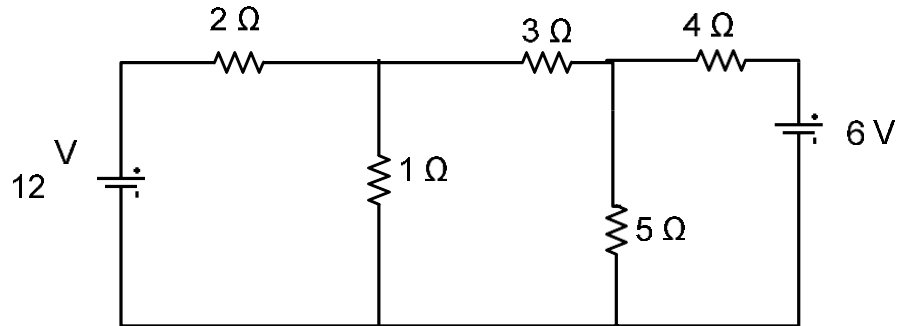
or,  $V_a \begin{bmatrix} 1 & 1 & 1 \\ R_1 & R_2 & R_3 \\ 1 & 2 & 3 \end{bmatrix} - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$

Hence,

- Node voltage multiplied by sum of all the conductance connected to this node. This term is positive
- The node voltage at the other end of each branch (connected to this node multiplied by conductance of this branch). This term is negative.

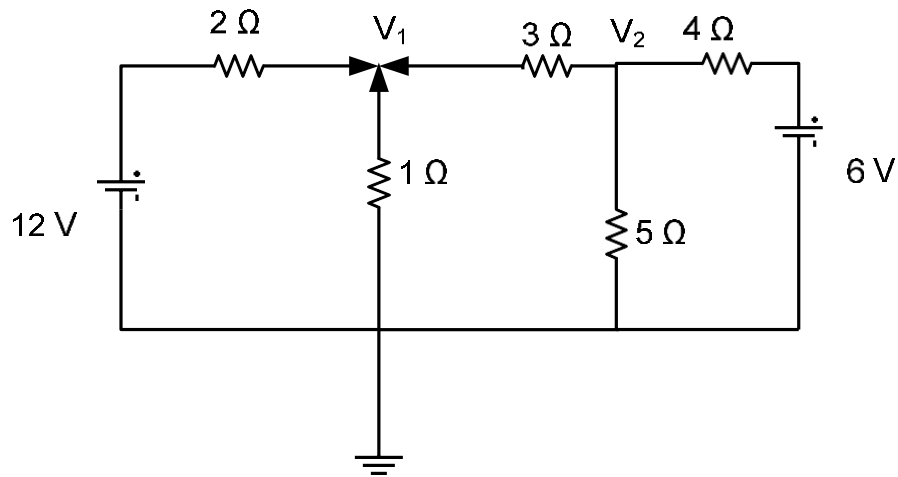
**Example:-** Use nodal analysis to find currents in the different branches of the circuit shown below.

(Supplementary Exam July- 2004)



Solution:-

Let  $V_1$  and  $V_2$  are the voltages of two nodes as shown in Fig below



Applying KCL to node-1, we get

$$\frac{12 - V_1}{2} + \frac{0 - V_1}{1} + \frac{V_2 - V_1}{3} = 0$$

$$\Rightarrow 36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$

$$\Rightarrow -11V_1 + 2V_2 = 36 \dots \dots \dots (1)$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 \dots\dots\dots(2)$$

Solving Eq (1) and (2) we get  $V_1 = 3.924$  Volt and  $V_2 = 3.584$  volt

$$\text{Current through } 2 \text{ } \Omega \text{ resistance} = \frac{12 - V_1}{2} = \frac{12 - 3.924}{2} = 4.038 \text{ A}$$

$$\text{Current through } 1 \text{ } \Omega \text{ resistance} = \frac{0 - V_1}{1} = -3.924 \text{ A}$$

$$\text{Current through } 3 \text{ } \Omega \text{ resistance} = \frac{V_1 - V_2}{3} = 0.1133 \text{ A}$$

$$\text{Current through } 5 \text{ } \Omega \text{ resistance} = \frac{0 - V_2}{5} = -0.7168 \text{ A}$$

$$\text{Current through } 4 \text{ } \Omega \text{ resistance} = \frac{6 - V_2}{4} = 0.604 \text{ A}$$

As currents through  $1\Omega$  and  $5\Omega$  are negative, so actually their directions are opposite to the assumptions.

#### **1.4. STAR-DELTA CONVERSION**

**Need:-** Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

In delta network, three resistors are connected in delta fashion ( $\Delta$ ) and in star network three resistors are connected in wye (Y) fashion.

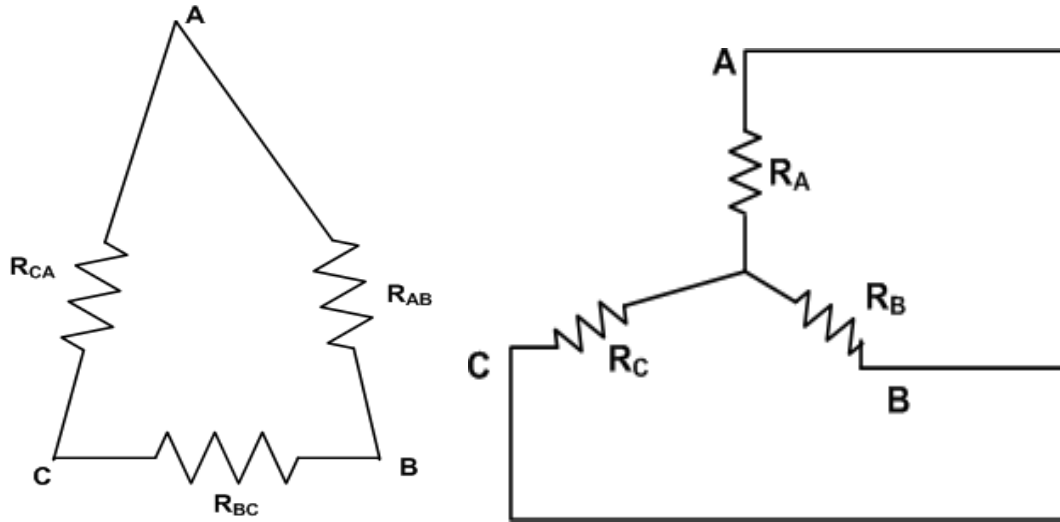


Fig. 1.4.1.

a) Delta connection

b) Star connection

1.4.1. **Delta to Star Conversion**:- From Fig. 1.4.1 (a),  $\Delta$  : Between A & B, there are two parallel path.

$$\text{Resistance between terminal A \& B} = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}}$$

From Fig. 1.4.1 (b), STAR: Between A & B two series resistances are there  $R_A + R_B$ . So, terminal resistances have to be the same.

$$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(1)$$

$$R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(2)$$

$$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(3)$$

Eq {(1)-(2)}+(3) & Solving,-

$$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(4)$$

$$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(5)$$

$$R_C = \frac{R_{CA} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \dots\dots\dots(6)$$

**Easy way to remember:-**

*Any arm of star connection =  $\frac{\text{Product of two adjacent arms of delta}}{\text{sum of arms of delta}}$*

**1.4.2. Star to Delta conversion**

Eq {(1) X (2)}+(2) X (3)+ (3) X (1) & Simplifying,-

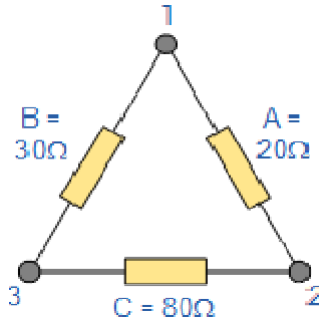
$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

**Easy way to remember:-** *Resistance between two terminals of delta = sum of star resistance connected to those terminals + product of the same to resistance divided by the third resistance.*

**Example(delta to star):-** Convert the following Delta Resistive Network into an equivalent Star Network.



$$Q = \frac{AC}{A+B+C} = \frac{20 \times 80}{130} = 12.31 \Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20 \times 30}{130} = 4.61 \Omega$$

$$R = \frac{BC}{A+B+C} = \frac{30 \times 80}{130} = 18.46 \Omega$$

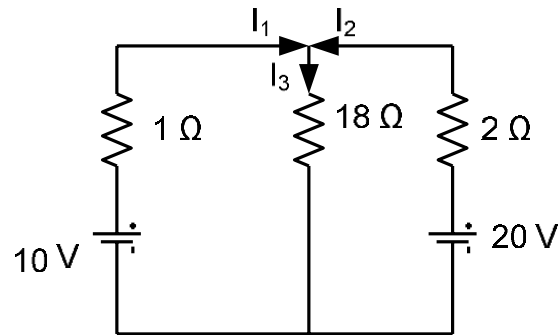
### NETWORK THEOREMS

- SUPERPOSITION THEOREM
- THEVENIN'S THEOREM
- NORTON'S THEOREM
- MAXIMUM POWER TRANSFER THEOREM

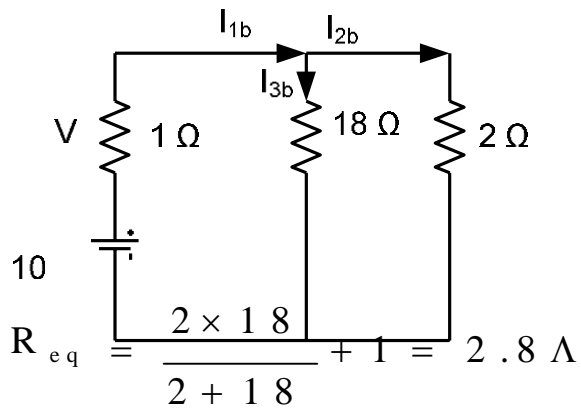
#### 1. Superposition theorem

Statement:- In a network of linear resistances containing more than one generator (or source of emf), the current which flows at any point is the sum of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistance.

**Example:-** By means of superposition theorem, calculate the currents in the network shown.



Step 1. Considering 10 V battery

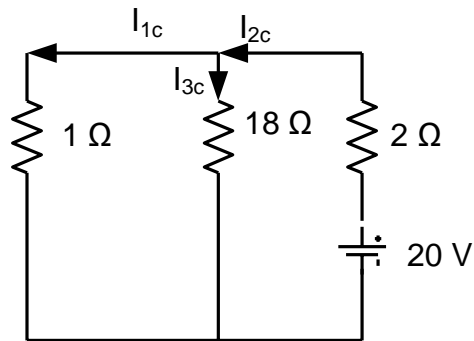


$$I_{1b} = \frac{10}{2.8} = 3.57 \text{ A}$$

$$I_{2b} = 3.57 \times \frac{18}{20} = 3.21 \text{ A}$$

$$I_{3b} = I_{1b} - I_{2b} = 0.36 \text{ A}$$

Step 2. Considering 20 V battery



$$R_{eq} = \frac{1 \times 18}{1 + 18} + 2 = 2.95 \Omega$$

$$I_{2c} = \frac{20}{2.95} = 6.78 \text{ A}$$

$$I_{1c} = 6.78 \times \frac{18}{19} = 6.42 \text{ A}$$

$$I_{3b} = I_{2c} - I_{1c} = 0.36 \text{ A}$$

Step 3. Results

$$I_1 = I_{1b} - I_{1c} = 3.57 - 6.42 = -2.85 \text{ A}$$

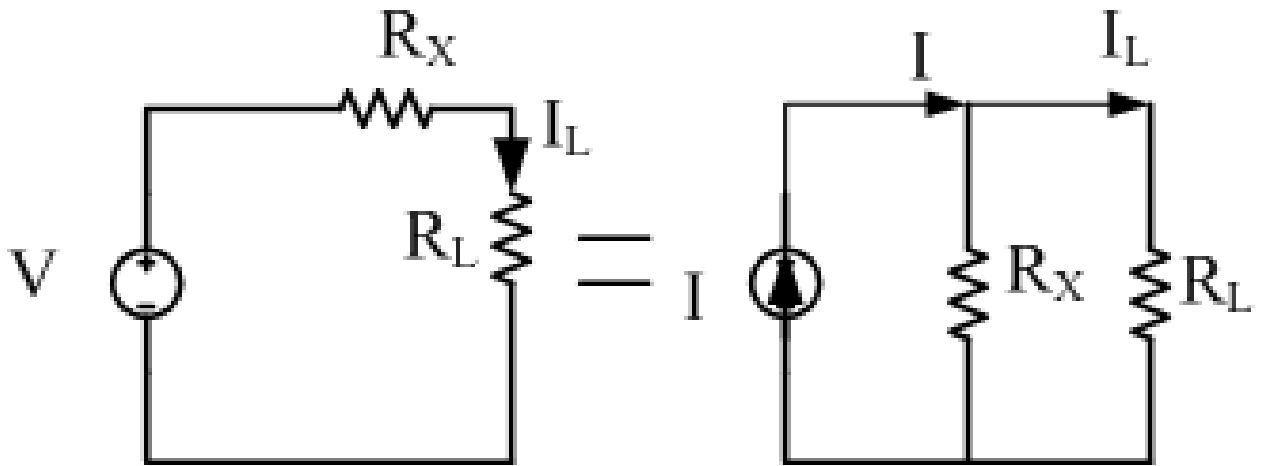
$$I_2 = I_{2c} - I_{2b} = 6.78 - 3.21 = 3.57 \text{ A}$$

$$I_3 = I_{3b} + I_{3c} = 0.36 + 0.36 = 0.72 \text{ A}$$

## 2. SOURCE CONVERSION:-

**Statement:** A voltage source (V) with a series resistance (R) can be converted to a current source ( $I=V/R$ ) with a parallel resistance (R) and vice-versa.

**Proof:-**



$$I_L = \frac{V}{R_X + R_L} \quad (1)$$

$$I_L = I \frac{R_X}{R_X + R_L} \quad (2)$$

From Eq. (1) & (2)

$$V = IR_X \quad (3)$$

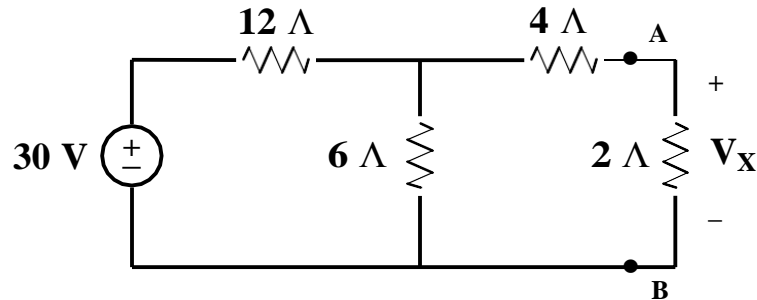
- **STATEMENT:** The two circuits are said to be electrically equivalent if they supply equal load currents with the same resistance connected across their terminals.
- voltage source having a voltage  $V$  and source resistance  $R_X$  can be replaced by  $I (= V/R_X)$  and a source resistance  $R_X$  in parallel with current source.
- Current source  $I$  and source resistance  $R_X$  can be replaced by a voltage source  $V (= IR_X)$  and a source resistance  $R_X$  in series with  $V$ .

### 3. **THEVENIN'S THEOREM:-**

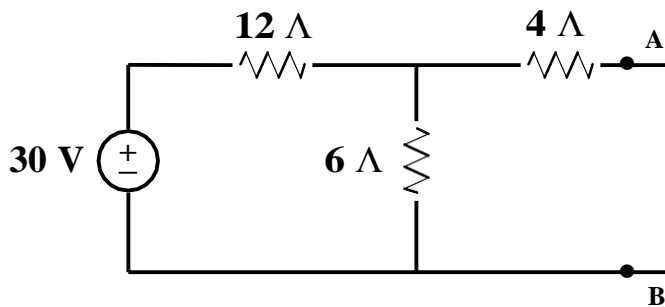
**Statement:-** Any pair of terminals  $AB$  of a linear active network may be replaced by an equivalent voltage source in series with an equivalent

resistance  $R_{th}$ . The value of  $V_{th}$  (called the Thevenin's voltage) is equal to potential difference between the terminals  $AB$  when they are open circuited, and  $R_{th}$  is the equivalent resistance looking into the network at  $AB$  with the independent active sources set to zero i.e with all the independent voltage sources short-circuited and all the independent current sources open-circuited.

Example:- Find  $V_X$  by first finding  $V_{TH}$  and  $R_{TH}$  to the left of A-B



Solution:- step1. First remove everything to the right of A-B.

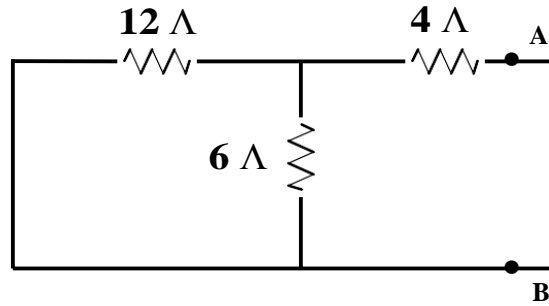


$$V_{AB} = \frac{(30)(6)}{6 + 12} = 10V$$

Notice that there is no current flowing in the  $4\ \Omega$  resistor (A-B) is open. Thus there can be no voltage across the resistor.

Step 2. To find  $R_{th}$

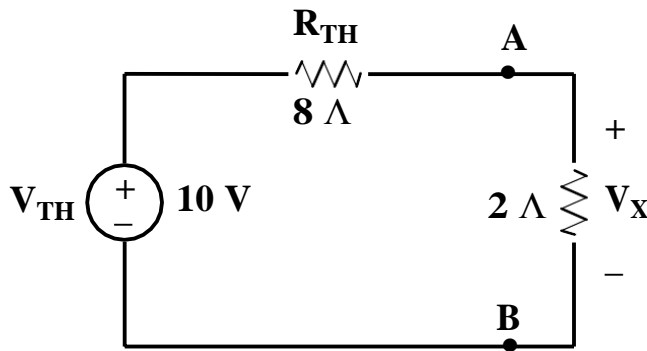
We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.



$$R_{TH} = 12 || 6 + 4 = 8 \Omega$$

Step 3. To find  $V_x$

After having found the Thevenin circuit, we connect this to the load in order to find  $V_x$ .



$$V_x = \frac{(10)(2)}{2+8} = 2V$$

#### 4. NORTON'S THEOREM:

**Statement:** Any two terminal linear active network (containing independent voltage and current sources), may be replaced by a constant current source  $I_N$  in parallel with a resistance  $R_N$ , where  $I_N$  is the current flowing through a short circuit placed across the terminals and  $R_N$  is the equivalent resistance of the network as seen from the two terminals with all sources replaced by their internal resistance.

**Example:** Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 50  $\Omega$  resistor.

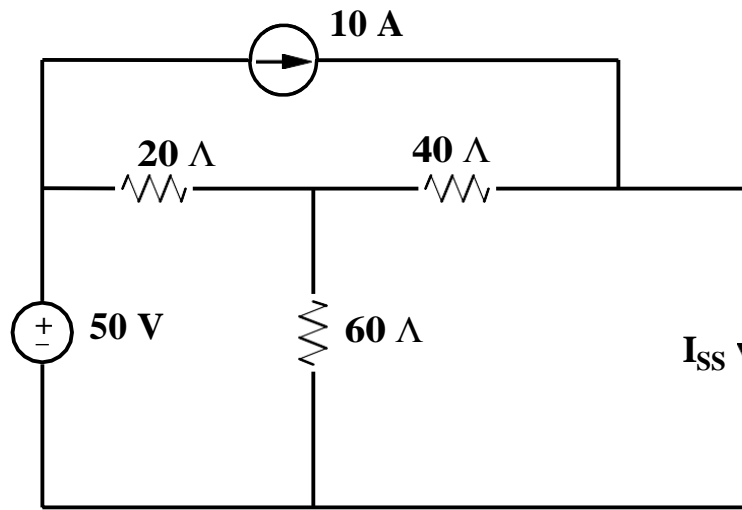
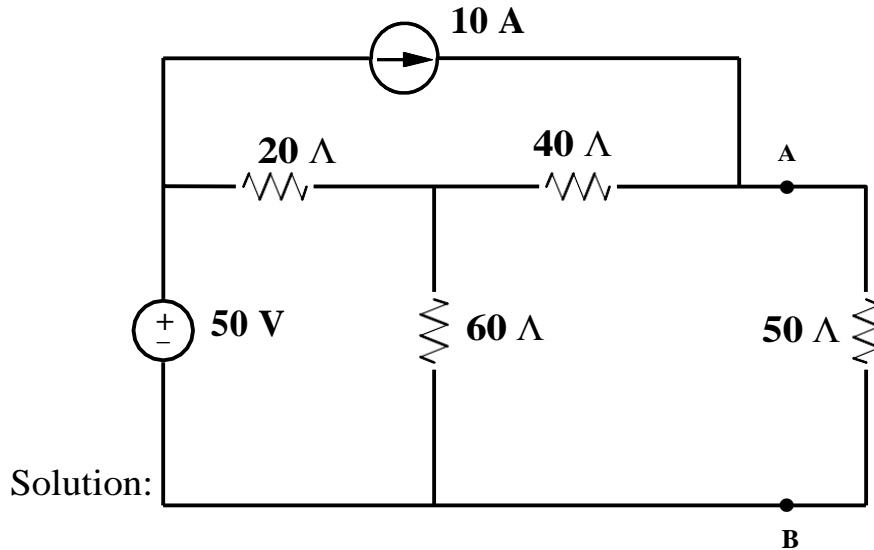
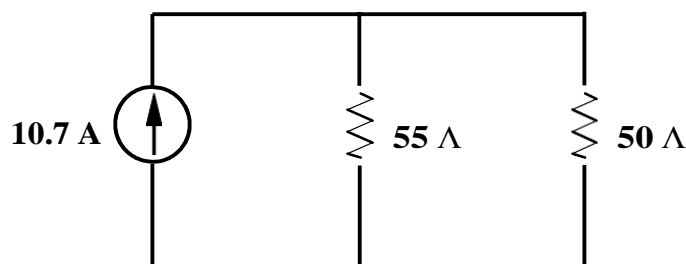


Fig. Circuit to find  $I_{NORTON}$

$$I_{SS} = 10.7 \text{ A}$$

It can also be shown that by deactivating the sources, we find the resistance looking into terminals A-B is  $R_N = 55 \Omega$

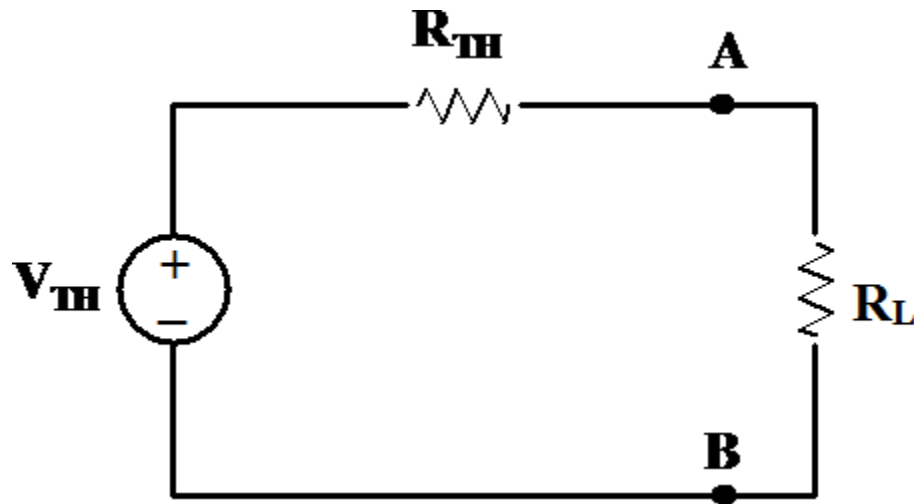
$R_N$  and  $R_{TH}$  will always be the same value for a given circuit. The Norton equivalent circuit tied to the load is shown below.



## 5. MAXIMUM POWER TRANSFER THEOREM:

- **Statement:** For any power source, the maximum power transferred from the power source to the load is when the resistance of the load  $R_L$  is equal to the equivalent or input resistance of the power source ( $R_{in} = R_{Th}$  or  $R_N$ ).
- The process used to make  $R_L = R_{in}$  is called impedance matching.

**Explanation:**



$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2}$$

$$\text{For } P_L \text{ to be maximum, } \frac{dP_L}{dR_L} = 0$$

$$\text{Or, } R_L = R_{TH}$$

$$\text{So, Maximum power drawn by } R_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$$

$$\text{Power supplied by the source} = \frac{V_{TH}^2}{(R_{TH} + R_L)}$$

## TRANSIENTS

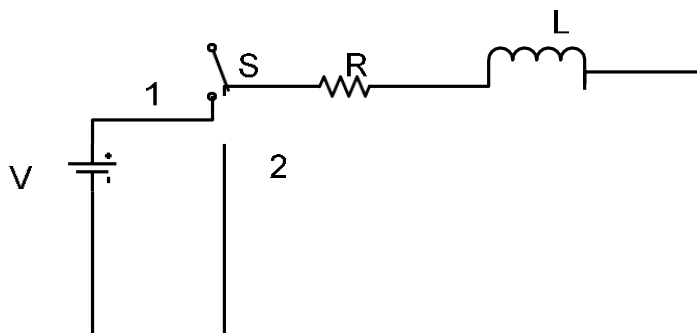
**Statement:** Sudden change in electrical circuit.

- Amplitude dies out and frequency is more
- Transient disturbances are produced whenever:-
  - An apparatus or circuit is suddenly connected to or disconnected from the supply.
  - A circuit is shorted
  - There is a sudden change in the applied voltage from one finite value to another.
  - *Transients are produced whenever different circuits are suddenly switched on or off from the supply voltage.*

Resultant current consists of two parts:-

- A final steady state or normal current
- A transient current superimposed on the steady state current
- Transient currents are not driven by any part of the applied voltage but are entirely associated with changes in the stored energy in inductors and capacitors.
- Since there is no stored energy in resistors, there are no transients in pure resistive circuit.

**Transient in R-L Series circuit:-**



When Switch 'S' is connected to '1',

$$V = R i + L \frac{d i}{d t}$$

$$i = i_{ss} + i_{tr}$$

$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = R i + L \frac{d i}{d t} = 0$$

$$\frac{d i}{d t} + \frac{R}{L} i = 0$$

$$\frac{d i}{d t} = -\frac{R}{L} i$$

$$\frac{d i}{i} = -\frac{R}{L} d t$$

$$\ln i = -\frac{R}{L} t; i_{tr} = K e^{-\frac{R}{L} t}$$

$$i_{ss} = \frac{V}{R}$$

$$i_{tr} = K e^{-\frac{R}{L} t}$$

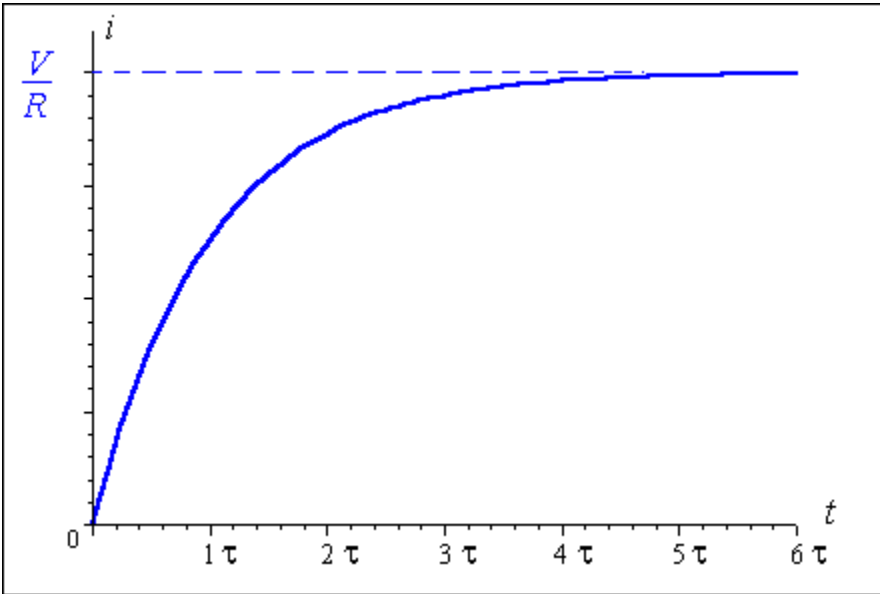
$$i = \frac{V}{R} + K e^{-\frac{R}{L} t}$$

At  $t=0, i=0$  So,

$$0 = \frac{V}{R} + K$$

$$K = -\frac{V}{R}$$

$$i = \frac{V}{R} \left( 1 - e^{-\frac{R}{L} t} \right)$$



$\lambda = \frac{L}{R}$  is called time constant and  $\frac{R}{L}$  is called damping coefficient of the circuit

$$V_R = iR = V \left( 1 - e^{-\frac{t}{\lambda}} \right)$$

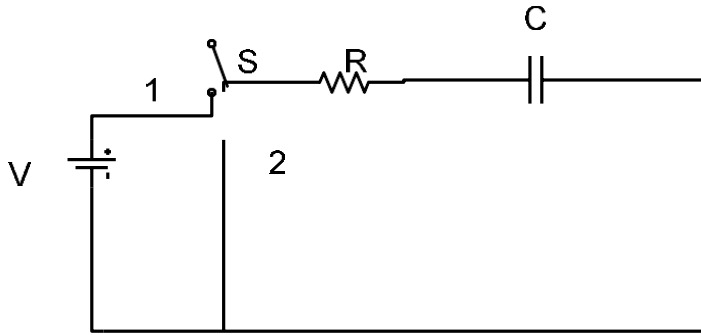
Emf of self inductance is  $-L \frac{di}{dt} = i R$

If  $t = \lambda$ , then  $i_t = I_0 e^{-1} = I_0 / e = I_0 / 2.718 = 0.37 I_0$

Hence, time period of a circuit is the time during which the transient current decrease to 0.37 of its initial value.

### **Transient in R-C Series Circuit:**

Consider an ac circuit containing a resistor of resistance R ohms and a capacitor of capacitance C farad across an a.c source of rms voltage V volts as shown in Fig. below:-



### Charging of RC

$$V = V_R + V_C$$

When switch is connected to '1' (charging):-

$$V = V_R + V_c$$

$$V = iR + \frac{1}{C} \int i dt$$

$$R \frac{di}{dt} + \frac{i}{C} = 0$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

$$i = K e^{-\frac{t}{RC}}$$

$$\text{At } t = 0^+ ; i = I_0 e^{-\frac{t}{\tau}}$$

$$K = \frac{V}{R} ; \text{So, } i = \frac{V}{R} e^{-\frac{t}{RC}}$$

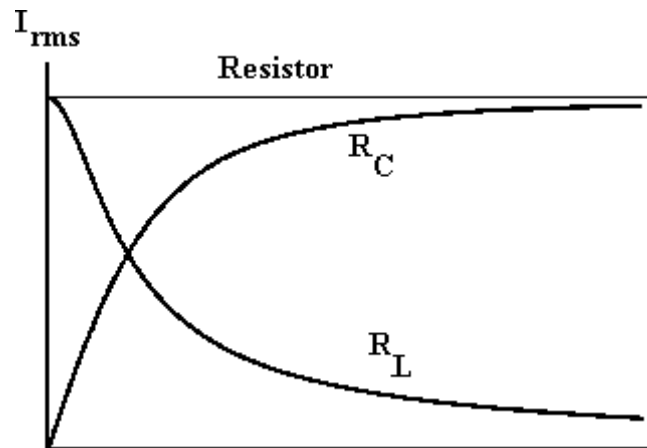
$$V_R = iR = I_0 e^{-\frac{t}{\tau}} R = V e^{-\frac{t}{\tau}}$$

$$V_c = \frac{1}{C} \int i dt = \frac{1}{C} \int_0^t I_0 e^{-\frac{t}{\tau}}$$

$$V_c = \frac{1}{C} I_0 (-\tau) \left[ \frac{-t}{\tau} \right]_0^t = \frac{1}{C} \frac{V}{R} (-RC)$$

$$V_c = -V \left( e^{-\frac{t}{\tau}} - e_0 \right)$$

$$V_c = V \left( 1 - e^{-\frac{t}{\tau}} \right)$$



### Discharging of RC

When connected to '2' in the Fig. above,

$$R i + \frac{1}{C} \int i dt = 0$$

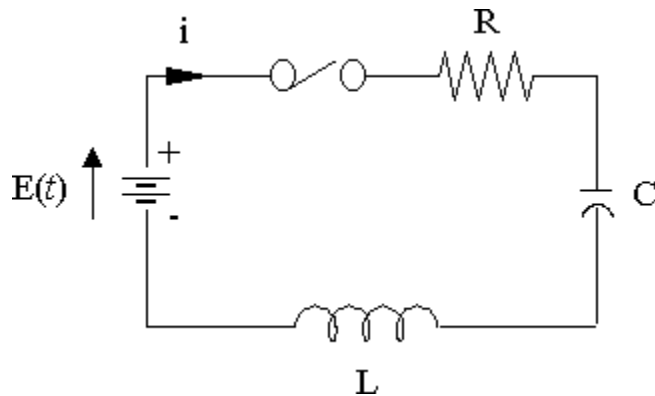
$$R \frac{di}{dt} + \frac{i}{C} = 0$$

$$i = K e^{\frac{-t}{RC}}$$

At  $t=0; i = \frac{-V}{R}$  (voltage across capacitor starts discharging in opposite direction to the original current direction)

$$i = -I_0 e^{\frac{-t}{RC}} = -I_0 e^{\frac{-t}{\tau}}$$

## Transient in R-L-C Series Circuit



- Two types of energy:- Electromagnetic and electrostatic. So any sudden change in the conditions of the circuit involves redistribution of these two energies.
- Transient current produced due to this redistribution may be unidirectional and decaying oscillatory.

From the above Fig,

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = V$$

$$R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\alpha \pm \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha \pm \beta = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\alpha \pm \beta = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\alpha = \frac{-R}{2L} \quad \text{and} \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Solution of differential equation is:-

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$\text{Roots are: } \alpha + \beta = P_1; \alpha - \beta = P_2$$

$K_1$  &  $K_2$  depends on boundary condition

Case 1: High loss circuit:-  $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$  i.e Overdamped

In this case,  $\beta$  is positive real quantity. Hence  $P_1$  and  $P_2$  are real but unequal.

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$i = K_1 e^{(\alpha + \beta)t} + K_2 e^{(\alpha - \beta)t}$$

$$i = K_1 e^{\alpha t} e^{\beta t} + K_2 e^{\alpha t} e^{-\beta t}$$

$$i = e^{\alpha t} \left[ K_1 e^{\beta t} + K_2 e^{-\beta t} \right]$$

The expression of 'i' is over damped transient non-oscillatory current.

CASE 2:- Low-loss circuit:  $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$  i.e Underdamped

In this case,  $\beta$  is imaginary. Hence roots are complex conjugate

$$P_1 = \alpha + j\beta; P_2 = \alpha - j\beta$$

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

$$i = K_1 e^{(\alpha + j\beta)t} + K_2 e^{(\alpha - j\beta)t}$$

$$i = K_1 e^{\alpha t} e^{j\beta t} + K_2 e^{\alpha t} e^{-j\beta t}$$

$$i = e^{\alpha t} \left[ K_1 e^{j\beta t} + K_2 e^{-j\beta t} \right]$$

The expression of 'i' is damped oscillatory

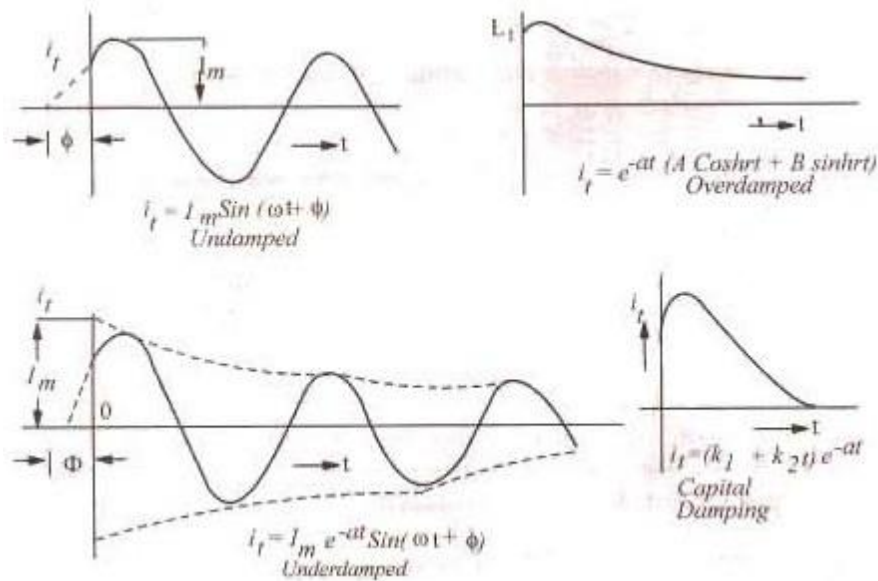
CASE 3:  $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$  i.e Critical damping

In this case  $\beta = 0$ , Hence roots  $P_1$  &  $P_2$  are real and equal.

$$P_1 = \alpha + 0 = \alpha; P_2 = \alpha - 0 = \alpha$$

$$i = K_1 e^{\alpha t} + K_2 t e^{\alpha t}$$

The above expression is of critical damping because current is reduced to almost zero in the shortest possible time.



**Example:** A coil having a resistance of  $2\Omega$  and an inductance of  $1\text{ H}$  is switched on to a  $10\text{ V D.C}$  supply. Write down the expression of current  $i(t)$  in the coil as a function of time

Ans:  $R = 2\Omega$ ,  $L = 1\text{ H}$ ,  $V = 10\text{ V}$

Time constant  $(\tau) = L/R = 1/2 = 0.5\text{ sec}$

Steady current  $= V/R = 10/2 = 5\text{ A}$

$$i(t) = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right)$$

$$i(t) = 5 \left( 1 - e^{-\frac{t}{0.5}} \right) \text{ A}$$

